Most central or least central?

How much modeling decisions influence a node's centrality ranking in multiplex networks

Sude Tavassoli and Katharina Anna Zweig

Graph Theory and Complex Network Analysis
Computer Science Department
Kaiserslautern University of Technology

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1. Multiplex Networks
   - Degree Centrality in Multiplex networks
2. Modeling decisions
   - Different Normalization Strategies
   - Aggregation Strategies
3. The sensitivity of nodes to modeling decisions
   - European airlines data set
   - $\Delta norm$ and Δagg
   - Higgs Boson Tweets Network
   - Law Firm dataset

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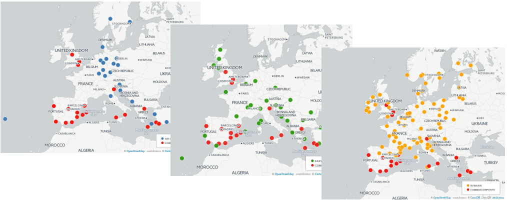
Analyzing the influence of nodes in networks –from simple graphs to multilayer networks– is always a fundamental question to be addressed in order to solve many real problems [6].

The usefulness of Centrality Measures in Multiplex Networks

- Analyzing flow processes in multiplex networks such as epidemic transmission in Transportation networks [2, 4].



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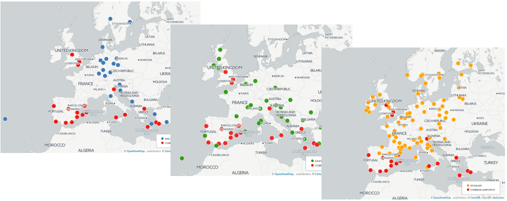
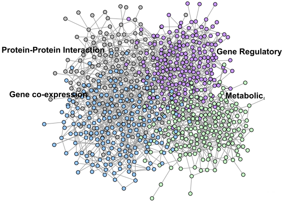
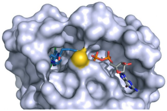
- Identifying cancer drivers in Biological networks using the representation of protein-protein interaction, gene regulation, co-expression, and metabolic network in a multiplex network [1].







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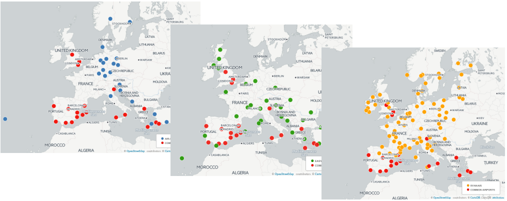

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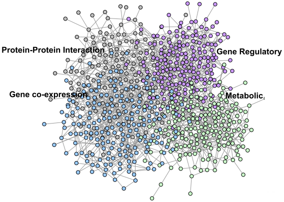
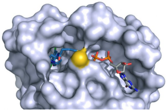
- Identifying cancer drivers in Biological networks using the representation of protein-protein interaction, gene regulation, co-expression, and metabolic network in a multiplex network [1].
- Analyzing leading drivers in Terrorist networks, where for instance, the importance of a node in "communication" layer is affected by the importance of the node in "trust" layer [6].





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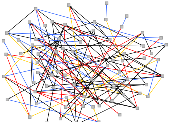

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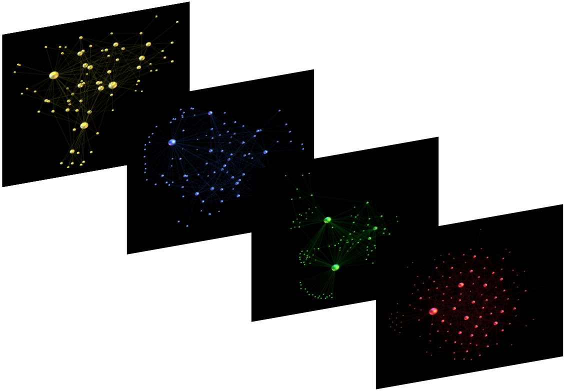
- A network with $|\mathcal{L}|$ layers $\mathcal{L} = \{L_1, L_2, \cdots, L_{|L|}\}$ where each layer l_i is a simple graph comprised of a set of V_i nodes and $E_i \subseteq V_i \times V_i$ edges.
- A set of nodes are common: $V^* = \bigcap_{i=1}^{|L|} V_i$.
- The degree $deg_i(v)$ of any node v is defined as the number of edges connected to the node v in layer L_i.
- The result of ranking is from position 1 to position $|V^*|$.



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These preprocessing seem to be inconsequential!?!?!?

Even the most simplest index using different modeling decisions can turn a node from the most central to the least central!

Outline

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NormMethod 1, for layer L_i takes $deg_i(v)$ for all $v \in V^*$ and normalizes it with the minimum and maximum values in the set of common nodes. This results in a vector of normalized indices of $[0, 1]$ for layer L_i.

$$\mathcal{C}_1(v, i) = \frac{deg_i(v) - min\{deg_i(v) | v \in V^*\}}{max\{deg_i(v) | v \in V^*\} - min\{deg_i(v) | v \in V^*\}}$$

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NormMethod 2 is similar to the last method but the normalization is done using the minimum and maximum values in the set of all nodes (V_i) in layer L_i.

$$\mathcal{C}_2(v,i) = \frac{deg_i(v) - min\{deg_i(v)|v \in V_i\}}{max\{deg_i(v)|v \in V_i\} - min\{deg_i(v)|v \in V_i\}}$$

NormMethod 3 uses the results by *NormMethod 2* and multiplies them with the fraction of the maximum degree in layer L_i and the maximum degree among all nodes in all $|\mathcal{L}|$ layers. This results in a vector of indices of nodes ($v \in V_i$) between $[0, \frac{\max\{deg_i(v)|v \in V_i\}}{\max\{deg_i(v)|v \in \bigcup V_j, 1 \leq i \leq |\mathcal{L}|\}}]$.

$$\mathcal{C}_3(v,i) = \mathcal{C}_2(v) \cdot \left(\frac{max\{deg_i(v)|v \in V_i\}}{max\{deg_i(v)|v \in \bigcup V_j, i \in [1, \ldots, |\mathcal{L}|]\}} \right)$$

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NormMethod 4 for each layer, we rank the nodes non-increasingly by their degree $deg_i(v)$ and obtain $r_i(v)$. This is then normalized by n_i.

$$\mathcal{C}_4(v,i) = \frac{r_i(v)}{n_i}$$

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Why the normalized ranking?

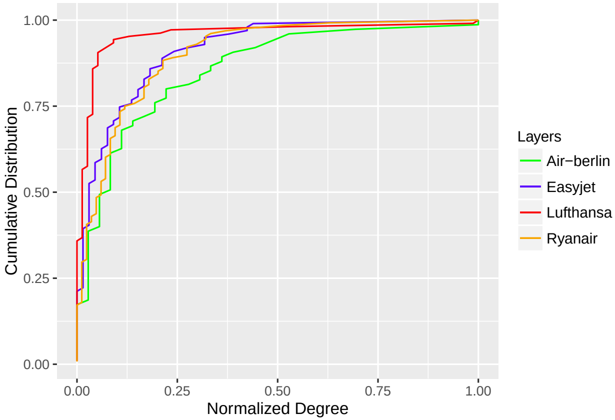

Figure: As can be seen, 90% of the degrees in Lufthansa are smaller than 70% of the degrees in Airberlin. If an aggregation wants to reward at least the most central nodes on each layer, this is difficult as even medium central nodes in Airberlin would get a larger index than most of the Lufthansa nodes.

Maximum Entropy Ordered Weighted Averaging (MEOWA) operator (denoted by λ) creates a single number based on the vector of a node's $|\mathcal{L}|$ normalized degrees as follows:

$$\lambda(C_x(v,1), C_x(v,2), \cdots, C_x(v,|\mathcal{L}|)) = \sum_j w_j \; d_j(v)$$

where $D = (b_1, b_2, ..., b_{|\mathcal{L}|})$ is the non-increasingly sorted vector of the normalized degrees, and w is a weight vector. The weight vector is obtained using the following function based on a parameter β [5]:

$$w_i = \frac{e^{\beta \frac{n-i}{n-1}}}{\sum_{j=1}^{n} e^{\beta \frac{n-j}{n-1}}}.$$

- $\beta = 20$: the weight vector is close to $(1, 0, \ldots, 0)$ and the aggregation strategy is (*OR*-operator); at least one layer.
- $\beta = -20$: the weight vector is close to $(0, 0, \ldots, 1)$ and the aggregation strategy is (*AND*-operator); all layers.
- $\beta = 0$: the weight vector is $(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})$ and the aggregation strategy is (*Average*).

Any β-value between the extreme strategies of "at least one" and "all layers" can be described using a set of proportional linguistic quantifiers (*a few*, *some*, *most*, *almost* introduced by Zadeh [9]).

$$\Omega = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) \frac{e^{\beta \frac{n-i}{n-1}}}{\sum_{j=1}^{n} e^{\beta \frac{n-j}{n-1}}}$$

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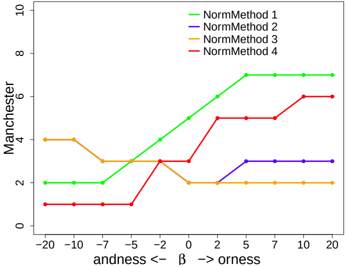
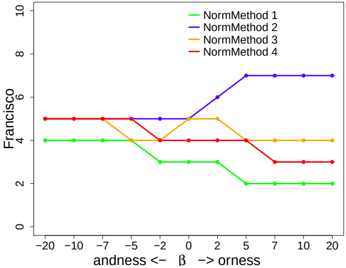
European airlines data set

A network comprised of four layers of airlines: Air Berlin, Easyjet, Lufthansa, and Ryan air. The order varies from 75 to 128 among four layers [2]. 9 nodes are common among the four layers.





Properties	Air-Berlin	Easyjet	Lufthansa	Ryanair
$\lvert V_i \rvert$	75	99	106	128
$\lvert E_i \rvert$	239	347	244	601
$\max_{v \in V_i}\{deg(v)\}$	37	67	78	85
$\max_{v \in V^*}\{deg(v)\}$	26	17	5	28
$\min_{v \in V_i}\{deg(v)\}$	1	1	1	1
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E.G.,

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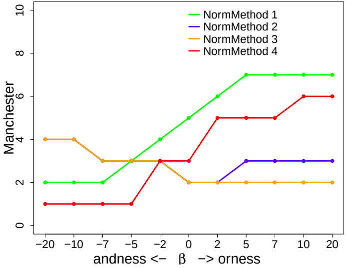

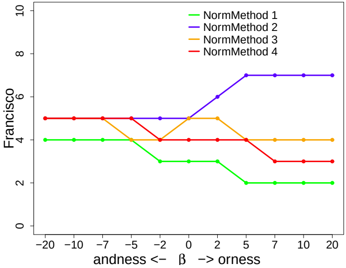

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$deg(Manchester)$: 1, 12, 5, 5 $\rightarrow \mathcal{C}_1(v)$: 0, 0.667, $\boxed{1}$, 0

$deg(Francisco)$: 12, 5, 1, 15 $\rightarrow \mathcal{C}_1(v)$: $\boxed{0.44}$, 0.2, 0, 0.435

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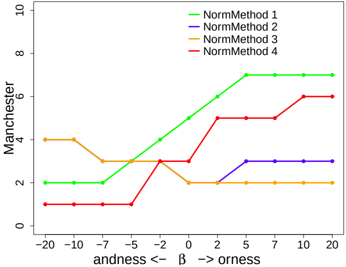

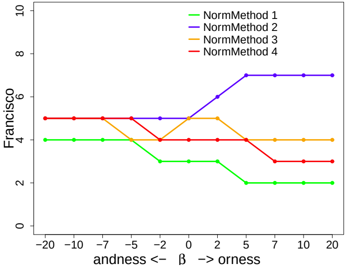

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$\mathcal{C}_2(v)$: 0, $\frac{11}{66}$, $\frac{4}{77}$, $\frac{4}{84}$ $\rightarrow$ 0, $\boxed{0.167}$, 0.052, 0.048

$deg(Francisco)$: 12, 5, 1, 15 $\rightarrow \mathcal{C}_1(v)$: $\boxed{0.44}$, 0.2, 0, 0.435

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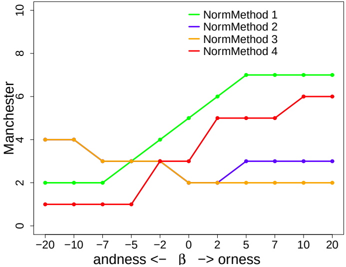

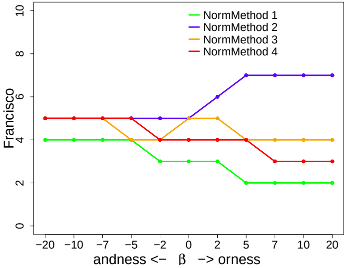

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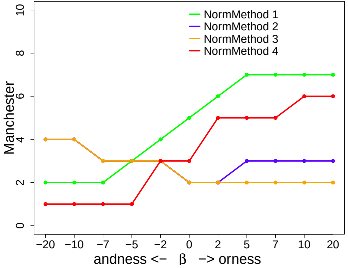

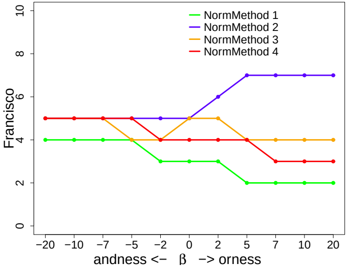

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$\mathcal{C}_4(v)$: 0.093, 0.818, $\boxed{0.887}$, 0.461

$deg(Francisco)$: 12, 5, 1, 15 $\rightarrow \mathcal{C}_1(v)$: $\boxed{0.44}$, 0.2, 0, 0.435

$\mathcal{C}_2(v)$: $\boxed{0.306}$, 0.061, 0, 0.167

$\mathcal{C}_3(v)$: 0.133, 0.048, 0, $\boxed{0.167}$

$\mathcal{C}_4(v)$: $\boxed{0.833}$, 0.611, 0.184, 0.789

The overall sensitivity of a node on the chosen normalization strategy is:

$$\Delta norm(v) := \max\{maxRank(v, \beta) - minRank(v, \beta) | \beta in \Gamma\}$$

where Γ is a set of different β-values.

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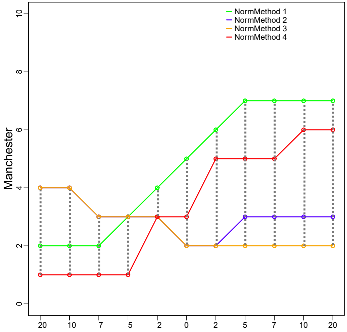
Example:

$\Delta norm(Manchester) := max\{3, 3, 2, 2, 1, 3, 4, 5, 5, 5, 5\} = 5$



Let $minRank(v, C_i)$ denote the minimal rank of node v based on normalization strategy C_i over all β-values and define $maxRank(v, C_i)$ accordingly.
The overall sensitivity of a node on the chosen aggregation strategy is:

$$\Delta agg(v) := \max\{maxRank(v, C_i) - minRank(v, C_i) | 1 \leq i \leq 4\}$$

Measuring the sensitivity of the nodes to the modeling decisions

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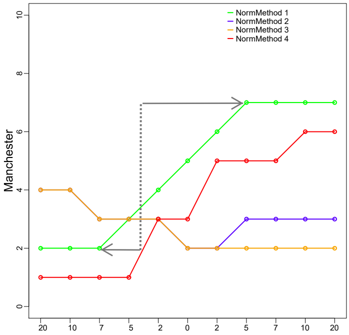
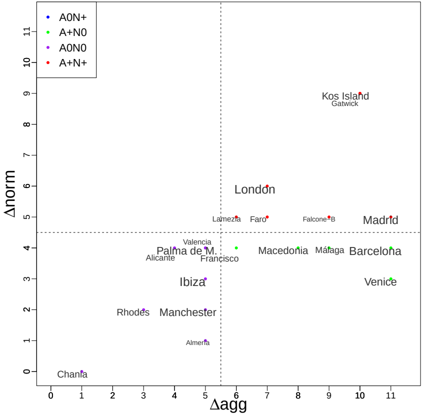
$\Delta agg(Manchester) := max\{5, 2, 2, 5\} = 5$



Now if we remove the layer of Lufthansa from the aggregation scenario, then we have 20 common nodes.



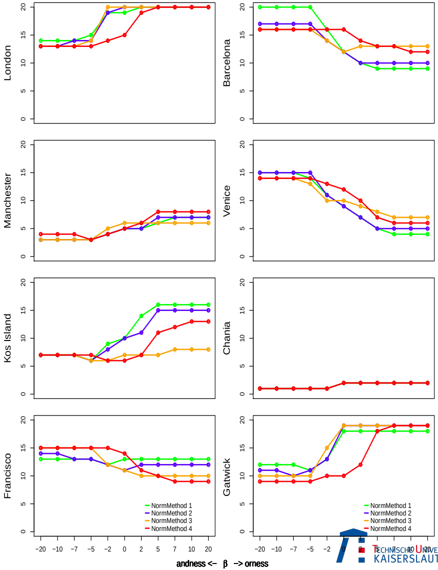


FIGURE: The sensitivity of the 20 common airports to the choice of aggregation strategy and normalization strategy. The four sections contain the nodes sensitive to only one choice ($A0N+$ or $A+N0$), those sensitive to none ($A0N0$), or both ($A+N+$).

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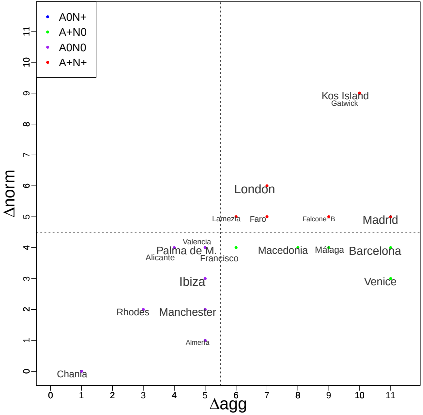

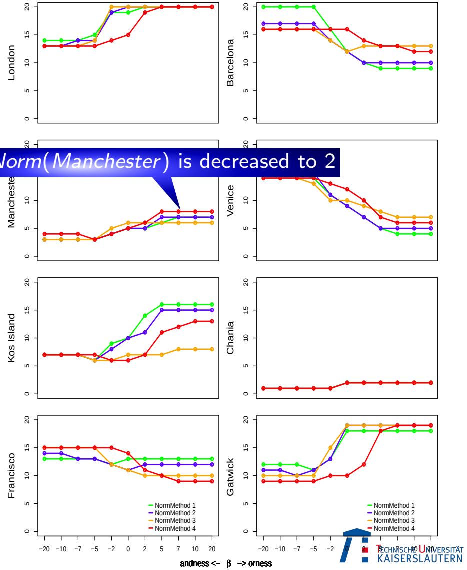

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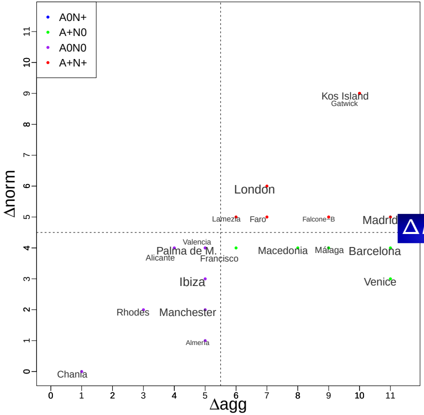

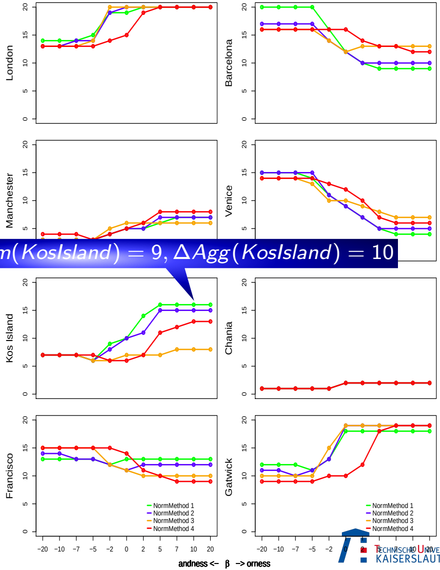

$\Delta Norm(KosIsland) = 9, \Delta Agg(KosIsland) = 10$

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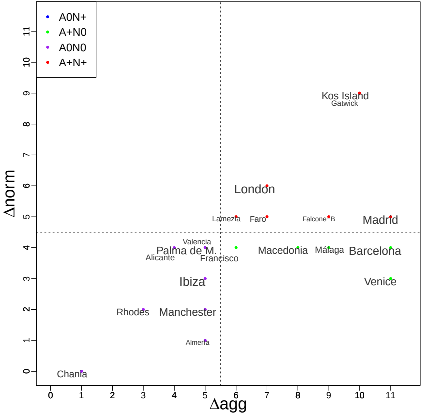
European airlines data set

Now if we remove the layer of Lufthansa from the aggregation scenario, then we have 20 common nodes.



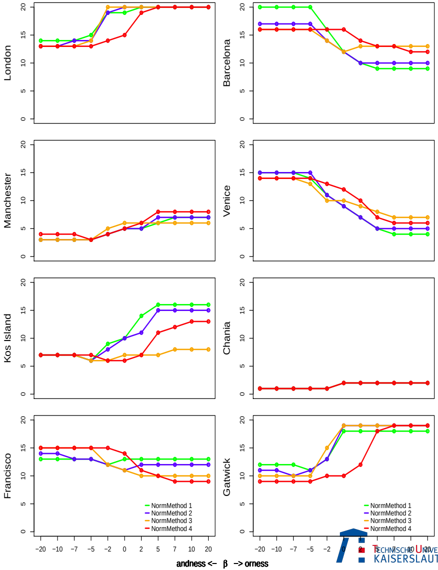


Figure: The sensitivity of the 20 common airports to the choice of aggregation strategy and normalization strategy. The four sections contain the nodes sensitive to only one choice ($A0N+$ or $A+N0$), those sensitive to none ($A0N0$), or both ($A+N+$).

Tweets dataset

A network comprised of four layers representing different interactions on topics concerning the "Higgs Boson": *mentioning*, *replying* to the tweets, *re-tweeting* the tweets of the other users, plus the social network of followers/followees [3].

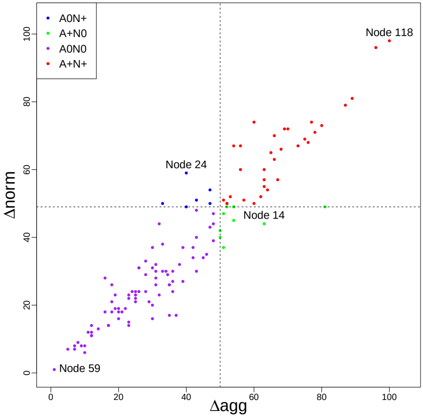


FIGURE: The sensitivity of 127 common nodes among four layers to the choices of aggregation (Δagg) and normalization ($\Delta norm$).

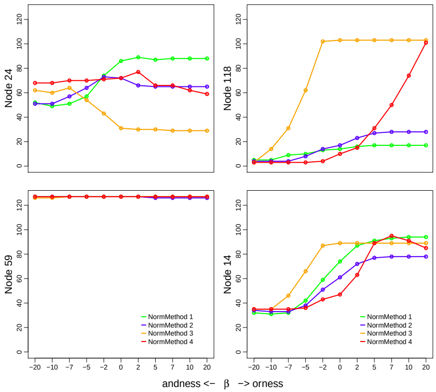


FIGURE: The ranking positions obtained using the different aggregation strategies (using the β parameter) for the aggregation of the four layers.

A network comprised of four layers representing different interactions on topics concerning the "Higgs Boson": *mentioning*, *replying* to the tweets, *re-tweeting* the tweets of the other users, plus the social network of followers/followees [3].

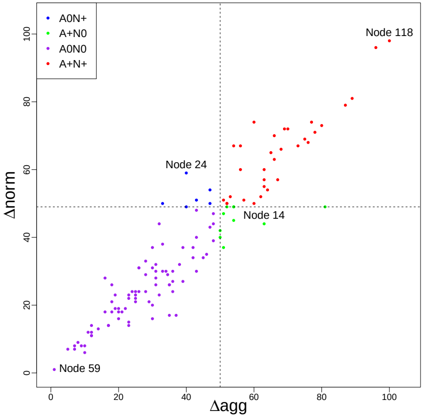


Figure: The sensitivity of 127 common nodes among four layers to the choices of aggregation (Δagg) and normalization ($\Delta norm$).

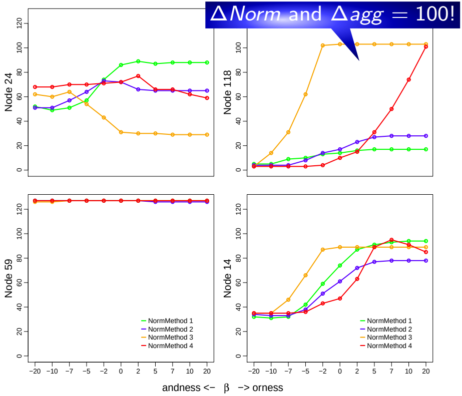


Figure: The ranking positions obtained using the different aggregation strategies (using the β parameter) for the aggregation of the four layers.

A network comprised of three layers of seeking advice, co-working, and having a friendship outside the firm among 71 attorneys [8].

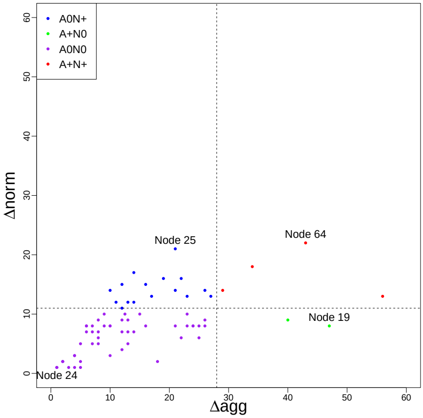


Figure: The sensitivity of 71 nodes to the choices of different aggregation strategies (Δagg) and the different normalization methods ($\Delta norm$).

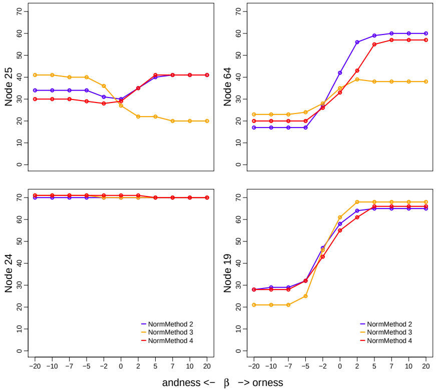


Figure: The rankings obtained using the different aggregation strategies (using the β parameter) for the aggregation of the results of three layers.

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- All these preprocessing steps need to be documented to make the analysis reproducible and its interpretation analyzable.

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- All these preprocessing steps need to be documented to make the analysis reproducible and its interpretation analyzable.
- Future works: categorizing nodes using fuzzy linguistic terms with respect to their overall centrality index.

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